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A short history of repetition-free words

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1 Introduction

The word "repetition" contains a 2-repetition (square) $titi = (ti)^2$ and a 3/2-repetition $epe = (ep)^{3/2}$. The word "homomorphism" contains a 5/2-repetition $omomo = (om)^{5/2}$, and "peeped" contains a 5/3-repetition $peepe = (pee)^{5/3}$.

It is Thue [43, 44] who first studied systematically repetition-free infinite words, but his pioneering works had been forgotten for a long time. Morse and Hedlund [28] developed the theory of symbolic dynamics without knowing his results. It was 60 years later when Hedland reported Thue's works in [19] (see Berstel [4]). The subject has become popular since the book by Laitare [24] was published. Berstel [5] gave a survey on the subject.

The words Thue constructed found important applications, for example, to the solution of the Burnside problem by Novikov [30], [31] (he used a result by Arson [2] without noticing Thue's works, see Adjan [1]) and the study of semigroup varieties (Burrie & Nelson [9], Sapir [39]).

2 X -free words

Let Σ be an alphabet (a finite set of letters), and let Σ^* be the free monoid generated by Σ . Σ^* is the set of finite words over Σ including the empty word 1. Furthermore, we consider the set Σ^ω of words of ω -words (one-sided infinite words). Set $\Sigma^\# = \Sigma^* \cup \Sigma^\omega$. For $x \in \Sigma^*$, x is a *subword* (or *factor*) if $y \in \Sigma^\#$ if $y = uxv$ ($u \in \Sigma^*, v \in \Sigma^\#$). Here if $u = 1$ (resp. $v = 1$), x is a *prefix* (resp. *suffix*) of y .

Define a distance δ on $\Sigma^\#$ as

$$\delta(x, y) = 2^{-\min\{n \mid a_n \neq b_n\}}.$$

for $x = a_1 \cdots a_n \cdots$ and $y = b_1 \cdots b_n \cdots$. It satisfies

$$\delta(x, y) \leq \max\{\delta(x, z), \delta(z, y)\}.$$

Proposition 2.1. (Σ^ω, δ) and $(\Sigma^\#, \delta)$ are compact totally disconnected metric spaces.

For $X \subset \Sigma^*$, $x \in \Sigma^\#$ is X -free (or *avoids* X), if any subword of x is not in X . A language L is X -free if any word in L is X -free. Let $L(\Sigma, X)$ be the language of X -free words and $L^\omega(\Sigma, X)$ be the set of X -free ω -words over Σ . Set $L^\#(\Sigma, X) = L(\Sigma, X) \cup L^\omega(\Sigma, X)$.

Proposition 2.2. $L^\#(\Sigma, X)$ is the closure of $L(\Sigma, X)$ in $\Sigma^\#$, and $L^\omega(\Sigma, X)$ is the set of limit points of $L(\Sigma, X)$.

Corollary 2.3. $L^\omega(\Sigma, X)$ is nonempty if and only if $L(\Sigma, X)$ is infinite.

As easily seen, $L^\omega(\Sigma, X)$ is perfect (there is no isolated points) if and only if any prefix of an ω -word $x \in L^\omega(\Sigma, X)$ is a prefix of two distinct ω -words in $L^\omega(\Sigma, X)$. If $L^\omega(\Sigma, X)$ is perfect, then it is homeomorphic to the Cantor ternary set and is uncountable.

We define

$$L(n) = L(\Sigma, X; n) = L(\Sigma, X) \cap \Sigma^n$$

and

$$d(n) = d(\Sigma, X; n) = |L(\Sigma, X; n)|.$$

Lemma 2.4. We have

$$d(n+m) \leq d(n) \cdot d(m)$$

for $m, n \in \mathbb{N}$.

Proposition 2.5 (see Kobayashi [22]). The limit $\mu = \mu(\Sigma, X) = \lim_{n \rightarrow \infty} d(n)^{1/n}$ exists, and it equals $\inf_n d(n)^{1/n}$. Either $\mu = 0$ or $1 \leq \mu \leq |\Sigma|$ holds.

We say $L(\Sigma, X)$ *grows exponentially* if there is $C > 1$ such that $d(n) \geq C^n$, and $L(\Sigma, X)$ *grows polynomially* if there is a polynomial p such that $d(n) \leq p(n)$. X is *avoidable* on Σ , if $L(\Sigma, X)$ is infinite, otherwise it is *unavoidable*.

Proposition 2.6. (1) $\mu = 0$ if and only if $L(\Sigma, X)$ is finite.

(2) $\mu > 1$ if and only if $L(\Sigma, X)$ grows exponentially.

We call μ the *growth rate*, *complexity* or *entropy* of $L(\Sigma, X)$.

3 Avoiding a finite set of words

Let X be a finite subset of Σ^* . Let $L = L(\Sigma, X)$, $\ell = \max\{|x| \mid x \in X\}$ and $V = L \cap \Sigma^{\ell-1} = \{v_1, \dots, v_s\}$. Define the *characteristic matrix* $M = (m_{ij})$ of X by

$$m_{ij} = \begin{cases} 1 & \text{if } v_j \text{ is a suffix of } v_i a \in L \text{ for some } a \in \Sigma \\ 0 & \text{otherwise.} \end{cases}$$

Lemma 3.1. For $\ell \geq 0$, $d(n + \ell - 1)$ is the number of paths of length n in the graph with adjacent matrix M .

Theorem 3.2. *The growth rate $\mu = \mu(\Sigma, X)$ is equal to the Frobenius root (the largest real eigenvalue) of M , and*

- (1) *if $\mu = 1$, L grows polynomially and L^ω is finite,*
- (2) *if $\mu > 1$, L grows exponentially and L^ω is perfect.*

Corollary 3.3. *For a finite set X , it is decidable whether X is unavoidable, L grows polynomially, or L grows exponentially.*

Example 3.4. Let $\Sigma = \{a, b\}$. $X_1 = \{aa, ab, bb\}$, $X_2 = \{aa, ab\}$ and $X_3 = \{aa\}$. Then, X_1 is unavoidable, $L(X_2, \Sigma)$ grows polynomially, $L(X_3, \Sigma)$ grows exponentially. The graphs associated with them are shown as follows respectively:

$$(1) \ a \leftarrow b \quad (2) \ a \leftarrow b \curvearrowright \quad (3) \ a \rightleftarrows b \curvearrowright$$

The following gives a way to give a good upper bound of $\mu(\Sigma, X)$ for an infinite X (see Shur 2008 [40]).

Theorem 3.5. *Let X be an infinite subset of Σ^* . Let $X_n = X \cap \Sigma^{\leq n} = \{x \in X \mid |x| \leq n\}$ and $\mu_n = \mu(\Sigma, X_n)$. Then, the sequence $\{\mu_n\}$ is decreasing and converges to $\mu(\Sigma, X)$.*

4 Unavoidable Patterns

Let V be an alphabet disjoint with Σ . A word p in V^* is called a *pattern*. An *instance* of p is a word in Σ^* obtained by substituting every variable in p by a nonempty word in Σ^* (see Bean, Ehrenfeucht & McNulty 1979 [3]). For a set P of patterns, $x \in \Sigma^*$ is *P -free* (or *avoids P*), if x is free from any instance of a pattern in P . Let $L(\Sigma, P)$ denote the set of P -free words over Σ and $L^\omega(\Sigma, P)$ be the set of P -free ω -words over Σ .

Example 4.1. (1) For $P_n = \{u^n\}$, $u \in V$, a P_n -free word is *n -power free* (square-free if $n = 2$, cube-free if $n = 3$).

(2) For $Q = \{u^3, uvuvu\}$, $u, v \in V$, a Q -free word is nothing but a *overlap-free* word.

P is *unavoidable* on Σ , if the set of the instances of patterns in P is unavoidable, that is, $L(\Sigma, P)$ is finite. P is *absolutely unavoidable* if it is unavoidable on any (finite) alphabet Σ .

Example 4.2. (1) The square u^2 is unavoidable on the two-letter alphabet, but it is not absolutely unavoidable.

(2) The pattern uvu is absolutely unavoidable.

The *adjacency graph* $\mathcal{A}(p)$ of $p \in V^*$ is the bipartite graph $(V^\ell \cup V^r, E)$, where $V^\ell \cup V^r$ is the union of two copies of V and $(u^\ell, v^r) \in E$ if uv appears in p . A *free set* F is a subset of V such that there is no path in $\mathcal{A}(p)$ from u^ℓ to v^r for any $u, v \in F$. A pattern p *reduces* to a pattern q (denoted as $p \Rightarrow q$), if q is obtained from p removing all letters in some free set.

Theorem 4.3 (Zimin 1982 [45], see Lothaire 2003 [25]). *A pattern p is absolutely unavoidable if and only if it is reduced to 1 using a finite number of reductions.*

Example 4.4. The pattern $uvwuvu$ is absolutely unavoidable because

$$uvwuvu \Rightarrow vuv \Rightarrow w \Rightarrow 1.$$

Corollary 4.5. *It is decidable whether a given pattern is absolutely unavoidable.*

5 Repetition-free words and morphisms

Let x be a nonempty word in Σ^* , y a prefix of x , and $s \in \mathbb{N}$. In this situation the word $x^s y$ is called a t -repetition of x , where

$$t = s + |y|/|x|.$$

For $t \in \mathbb{R}$, $x \in \Sigma^\#$ is t -repetition-free if x contains no s -repetition with $s \geq t$, and x is *weakly t -repetition-free* if x contains no s -repetition with $s > t$.

To treat these two kinds of repetition-freeness commonly we introduce the ordered set

$$\overline{\mathbb{R}} := \mathbb{R} \cup \mathbb{Q}^+, \quad \mathbb{Q}^+ = \{t^+ \mid t \in \mathbb{Q}\},$$

in which $a < a^+ < b$ for $a \in \mathbb{Q}$, $b \in \mathbb{R}$ with $a < b$. For $\alpha \in \overline{\mathbb{R}}$, x is α -repetition-free if it has no t -repetition with $t \geq \alpha$ as subword. A 2^+ -repetition is an *overlap*.

Let $L(\Sigma, \alpha) = L(k, \alpha)$ denote the set of all α -repetition-free words over Σ with $|\Sigma| = k$. Define

$$d(\Sigma, \alpha; n) = d(k, \alpha; n) = |L(k, \alpha) \cap \Sigma^n|,$$

and

$$\mu(\Sigma, \alpha) = \mu(k, \alpha) = \lim_{n \rightarrow \infty} d(k, \alpha; n)^{1/n}.$$

Let Δ be another alphabet. A morphism $\Phi : \Sigma^* \rightarrow \Delta^*$ of monoids is *growing* if $|\Phi(a)| \geq 1$ for all $a \in \Sigma$, and $|\Phi(a)| \geq 2$ for some $a \in \Sigma$. Φ is *strictly growing* if $|\Phi(a)| \geq 2$ for all $a \in \Sigma$. Φ is *uniformly growing* if there is $p \geq 2$ such that $|\Phi(a)| = p$ for all $a \in \Sigma$. Φ is α -repetition preserving if $x \in \Sigma^*$ is an α -repetition, then so is $\Phi(x)$. Φ is α -repetition-free if $x \in \Sigma^*$ is α -repetition-free, then so is $\Phi(x)$. A nontrivial uniform morphism is α -repetition preserving for any α .

The existing of morphisms with above properties gives information about repetition-free words.

Theorem 5.1 (Thue 1906 [43], Kobayashi 1986 [22]). *Let $\alpha \in \overline{\mathbb{R}}$ and $\Phi : \Sigma^* \rightarrow \Sigma^*$ be a uniformly growing α -repetition-free morphism. Then, $L^\omega(\Sigma, \alpha)$ contains a nonempty perfect subset, in particular, $L^\omega(\Sigma, \alpha)$ is uncountable.*

Theorem 5.2 (Brandebberg 1983 [7]). *Suppose that $|\Sigma| < |\Delta|$ and $\Phi : \Delta^* \rightarrow \Sigma^*$ is a uniformly growing injective α -free morphism. If $L(\Sigma, \alpha) \neq \{1\}$, Then $L(\Sigma, \alpha)$ grows exponentially.*

Theorem 5.3 (Restivo & Salemi 1985 [35], Kobayashi 1986 [22]). *Let $\Phi : \Sigma^* \rightarrow \Sigma^*$ be a strictly growing α -repetition preserving morphism. If $\exists N > 0$ s.t. $\forall x \in L(\Sigma, \alpha)$, $\exists u, v, y \in \Sigma^*$ s.t. $|u|, |v| \leq N$, $x = u\Phi(y)v$. Then, $L(\Sigma, \alpha)$ grows polynomially.*

6 Binary words

The Thue morphism $\Theta : \{a, b\}^* \rightarrow \{a, b\}^*$ is defined by

$$\Theta(a) = ab, \Theta(b) = ba.$$

It produces the Thue words

$$a, \Theta(a) = ab, \Theta^2(a) = abba, \Theta^3(a) = abbabaab, \dots$$

Theorem 6.1 (Thue 1906 [43]). *Θ is overlap-free. So, the Thue words are overlap-free.*

Corollary 6.2. *$L^\omega(2, 2^+)$ contains a nonempty perfect set and uncountable.*

More strongly, we have

Theorem 6.3 (Fife 1983 [18]). *$L^\omega(2, 2^+)$ is perfect.*

Though $L^\omega(2, 2^+)$ is uncountable, $L(2, 2^+)$ grows very slowly.

Lemma 6.4. *For any $x \in L(2, 2^+)$, $\exists u, v, y \in \Sigma^*$ s.t.*

$$x = u\Theta(y)v, |v| \leq 2, |v| \leq 2.$$

Theorem 6.5 (Restivo & Salemi 1985 [35]). *$L(2, 2^+)$ grows polynomially.*

Though $L(2, 2^+)$ grows polynomially, $d(n) = |L(2, 2^+) \cap \Sigma^n|$ cannot be approximated by a single polynomial (see (3) below). The estimation of $d(n)$ has been improved as follows.

(1) Restivo & Salemi 1985 [35]: $d(n) \leq C \cdot n^{3.906\dots}$.

(2) Kobayashi 1988 [23]: $C_1 \cdot n^{1.155} < d(n) < C_2 \cdot n^{1.587}$.

(3) Cassaigne 1993 [11]: $\sigma^- < 1.276 < 1.332 < \sigma^+$, where

$$\sigma^- = \underline{\lim} \log d(n) / \log n, \sigma^+ = \overline{\lim} \log d(n) / \log n.$$

(4) Jungers, Protasov & Blondel 2009 [20]:

$$1.2690 < \sigma^- < 1.2736 < 1.3322 < \sigma^+ < 1.3326,$$

and $\log d(n) / \log n \rightarrow \sigma$ on a set of density 1 of n with $1.3005 < \sigma < 1.3098$.

Define a morphism $\beta : \{a, b, c\}^* \rightarrow \{a, b\}^*$ by

$$\beta(a) = aababb, \beta(b) = aabbab, \beta(c) = abbaab.$$

Theorem 6.6 (Brandenburg 1983 [7]). β is cube-free, and $L(2, 3)$ grows exponentially.

The estimation of $\mu(2, 3)$ has been improved as follows.

- (1) Brandenburg 1983 [7]: $1.08 < \mu(2, 3) < 1.522$.
- (2) Edlin 1999 [16]: $\mu < 1.4576$.
- (3) Shur 2008 [40] 2009 [41] 2010 [42]: $1.45757131 < \mu(2, 3) < 1.457577286$.
- (4) Shur 2009 [41]: $1.82109999323 < \mu(2, 4) < 1.8210999324$.

7 Repetition threshold

Define the *repetition threshold* $RT(r)$ and the *exponential repetition threshold* $ERT(r)$ for $r \geq 2$ by

$$RT(r) = \sup\{\alpha \in \overline{\mathbb{R}} \mid L^\omega(r, \alpha) = \emptyset\},$$

and

$$ERT(r) = \inf\{\alpha \in \overline{\mathbb{R}} \mid L(r, \alpha) \text{ grows exponentially}\}.$$

By Corollary 6.2, Corollary 6.5 and Theorem 6.6, we see

$$2 = RT(2) < ERT(2) \leq 3.$$

Theorem 7.1 (Karhmäki & Shallit 2004 [21]). $ERT(2) = 7/3 = 2.333\dots$. Moreover, $d(2, 7/3) \leq C \cdot n^{4.644}$ and $C_1 \cdot 1.011^n \leq d(2, 7^+/3) \leq C_2 \cdot 1.23^n$.

The estimation of $d(2, 7/3)$ has been improved as follows, where

$$\sigma^- = \underline{\lim} \log d(2, 7/3; n) / \log n, \quad \sigma^+ = \overline{\lim} \log d(2, 7/3; n) / \log n.$$

- (1) Karhmäki & Shallit 2004 [21]: $\sigma^+ < 4.644$.
- (2) Blondel, Cassaigne & Jungers 2009 [6]:

$$1.2690 < \sigma^- < 2.0035 < 2.0121 < \sigma^+ < 2.1050.$$

The estimation of $\mu(2, 7^+/3)$ has been developed as

- (1) Karhmäki & Shallit 2004 [21]: $1.011 < \mu(2, 7^+/3) < 1.23$.
- (2) Shur 2008 [40] 2009 [41]: $1.22062539 < \mu(2, 7^+/3) < 1.22064486$.

Define a morphism $\beta' : \{a, b, c, d\}^* \rightarrow \{a, b, c\}^*$ by

$$\beta'(a) = abacabcacbabcbabc,$$

$$\beta'(b) = abacabcacbacababc,$$

$$\beta'(c) = abacabcacbcabcbabc,$$

$$\beta'(d) = abacabcacabcbabc.$$

Theorem 7.2 (Brandenburg [7]). β' is square-free, and $L(3, 2)$ grows exponentially.

The estimation of $\mu(3, 2)$ has been improved as follows.

- (1) Brinkhuis 1983 [8]: $1.0293 < \mu(3, 2) < 1.316$.
- (2) Brandebberg 1983 [7]: $1.032 < \mu(3, 2) < 1.38$.
- (3) Richard & Grimm 2004 [37]: $\mu(3, 2) < 1.301762$.
- (4) Shur 2008 [40] 2009 [41]: $1.30175824 < \mu(3, 2) < 1.3017619138$.

About the perfectness of $L^\omega(r, \alpha)$ we have

Theorem 7.3 (Shelton 1981, 1982 [38]). $L^\omega(3, 2)$ is perfect.

Theorem 7.4. (1) (Currie & Shelton 1996 [14]) $L^\omega(r, \alpha)$ is perfect, if $1 < \alpha < 2$ and r is sufficiently large.

(2) (Mignosi, Restivo & Salemi 1995 [26]) $L^\omega(r, \alpha)$ is perfect, if $\alpha \geq 2$ and $r > (5 + \sqrt{5})/2 = 3.618\dots$

By Theorem 7.2 we see $1 < \text{RT}(3) \leq \text{ERT}(3) \leq 2$.

Define a morphism $\delta : \{a, b, c\}^* \rightarrow \{a, b, c\}^*$ by

$$\delta(a) = abcacbcabcbacbacba,$$

$$\delta(b) = bcabacabcbacabacb,$$

$$\delta(c) = cabcbabcbabcbabcbac.$$

Theorem 7.5 (Déjean 1972 [15]). δ is $7^+/4$ repetition-free, and

$$\text{RT}(3) = 7/4 = 1.75.$$

Conjecture 7.6 (Déjean). $\text{RT}(4) = 7/5$, and $\text{RT}(r) = r/(r - 1)$ for $r \geq 5$.

The conjecture has been finally proved to be true. The following is its history.

$r = 3$: Déjean 1972 [15],

$r = 4$: Pansiot 1984 [34],

$5 \leq k \leq 11$: Moulin-Ollagnier 1992 [29],

$12 \leq k \leq 14$: Mohammad-Moori & Currie 2007 [27],

$33 \leq k$: Carpi 2007 [10],

$27 \leq k$: Currie & Rampersad 2009 [12].

$8 \leq k \leq 38$: Rao 2011 [36], Currie & Rampersad 2011 [13].

Theorem 7.7 (Ochen 2006 [32]). $L(3, 7^+/4)$ and $L(4, 7^+/5)$ grow exponentially, that is, $\text{RT}(3) = \text{ERT}(3)$, $\text{RT}(4) = \text{ERT}(4)$.

Conjecture 7.8 (Ochen). $\text{RT}(r) = \text{ERT}(r)$ for all $r \geq 3$.

If this conjecture is true, the case $r = 2$ is very exceptional.

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